

# Survey on Calculation of Mutual Impedance of Planar Dipole Array Using Method of Moment

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**Abstract-** The Method of Moments (MOM) technique is a powerful numerical method for solving linear partial differential equations which are in the form of integral equations. This method gives the solution for the induced current which has been formulated as integral where the integrand is the current density(unknown). In this paper, the literature for the calculation of mutual impedance of a planar dipole antenna using MOM is quite fragmented. Here, we give a description on MOM for calculating the current distribution of the dipole antenna for a basis function. The formulas are derived from the Pocklington's integral equation using MOM.

**keywords-** dipole antenna; method of moments; integral equation; impedance

## I. INTRODUCTION

An antenna is defined as a “transmitting or receiving system that is designed to radiate or receive electromagnetic waves” [1]. The dipole antenna is the simplest and probably the most widely used class of antenna in radio and telecommunications. The most known one is the half wave dipole antenna, in that each of the two components is approximately 1/4th wavelength long, for that the whole antenna is a half-wavelength in length. This is a simple antenna to understand the theoretical explanation. Now, to work appropriately, the antenna must be matched to the transmitter system. The range of the antenna systems impedance of a few ohms to several thousand of ohms[7]. The magnetic field is surrounded through out the straight wire that has an AC current. With increasing in distance, the magnetic field from the dipole becomes weaker . The radiated magnetic field from an antenna is known as "electromagnetic radiation". The radiating antenna becomes the transmitting antenna. The second wire has the current induced into it & passes the electromagnetic waves. This second wire is the receiving antenna. The voltage generated in the receiving antenna is much weaker than the voltage in the transmitting antenna.

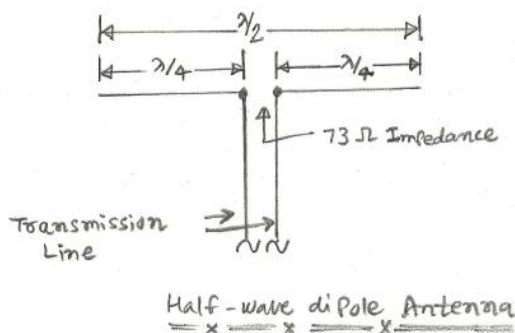


Figure 1.1 Basic diagram of dipole antenna

## II. POCKLINGTON’S INTEGRAL EQUATION (PIE)

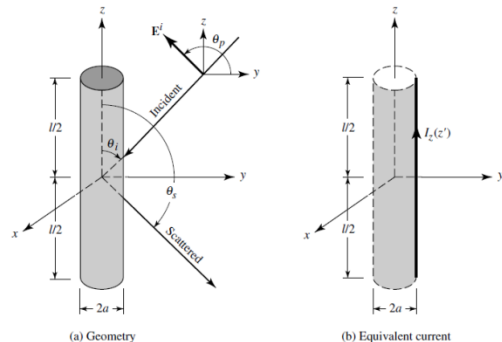


Figure 2.1 Dipole and its equivalent current

The Pocklington’s Integral Equation is a well-known formula for simple wire antennas. To derive Pocklington’s integral equation, refer to Figure 2.1 Although this derivation is general, it can be used either when the wire is a scatterer or an antenna. Figure 2.1 represents the dipole with its equivalent current[6]. In the Cartesian coordinate system the wire antenna is positioned. The current is directed along the z-axis & restricted on the surface of the wire antenna. Current segments are located at z’ coordinate. At coordinates z, field observation points are located. A feed gap is at z=0. At the surface of the antenna, the total tangential electric field is zero. The induced current density then reradiates and also an electric field is produced that is known as the scattered electric field. The Pocklington’s integral equation is stated as:[10]

$$j/\omega\epsilon \int_{-l/2}^{l/2} [k^2 + \frac{\partial^2}{\partial z'^2}] I_z(z') G(z, z') dz' = -\frac{V_i}{\Delta Z} \tag{1}$$

Here, G(z, z') is the Green's function and is given by  $\frac{e^{-jkR}}{4\pi R}$ ,

$$R = \sqrt{4(a \sin \frac{\theta}{2})^2 + (Z - Z')^2} \tag{4}[6]$$

and 'a' is the radius.

The current  $I_z(z')$  is described along the total length of the dipole wire from  $z' = -l/2$  to  $z' = l/2$ . The kernel  $[k^2 + \delta^2/\delta z'^2]$  denotes the differential operator on the free space Green’s function . The constant 'k' is the free space wave number.  $\Delta Z$  is the feed gap and  $V_i$  is the excitation function.  $\epsilon$  is the permittivity and  $\omega$  is the angular frequency.

## III. METHOD OF MOMENT

The solution procedure begins in terms of an orthogonal set of basic functions by defining the unknown current distribution. There are two categories of basic functions

exist i.e., sub domain basic functions, significantly popular in industry, it subdivides the wire into small segments and model the current waveform on each segment by a simple geometrical waveform, such as a rectangle, triangle or sinusoidal function. The amplitudes of these functions represent the expansion coefficients. These simple functions are illustrated in Figure 3.1.

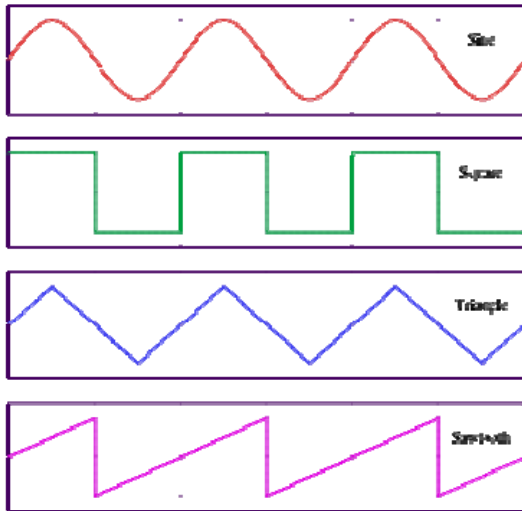


Fig 3.1: Four different types of basic functions in MOM

For the linear operational equation, the Method of moment can be applied expressed as;

$$L(f) = g \tag{2}$$

Where 'L' is the operator of linearity, which can be the differential operator, an integral or an integral-differential operator. 'f' is the unknown function and 'g' is the known function. In the 1st step of Method Of Moment, it is considered that 'f' can be defined over the domain of 'L',

$$f = \sum_{i=1}^N \alpha_i f_i \tag{3}$$

where  $\alpha_i$  are unknown scalar function and  $f_i$  is known expansion function . Now, for approximate solutions, it is necessary to mention that (3) is a finite summation whereas, it is generally an infinite one for exact solutions,. Using (2), (3) and linear property of L , we have;

$$g = \sum_{i=1}^N \alpha_i L(f_i) \tag{4}$$

Now, a set of linearly independent weighting functions  $\{w_1, w_2, \dots, w_N\}$  in the range of L is defined. Now , we took the inner product of (4) with each  $w_j$  and using the linear property gives,

$$\sum_{i=1}^N \alpha_i \langle w_j, L(f_i) \rangle = \langle w_j, g \rangle \tag{5} \text{ [4][5][7]}$$

The above equation may be written as:

$$\begin{pmatrix} \langle w_1, L(f_1) \rangle & \langle w_1, L(f_2) \rangle & \dots & \langle w_1, L(f_N) \rangle \\ \langle w_2, L(f_1) \rangle & \langle w_2, L(f_2) \rangle & \dots & \langle w_2, L(f_N) \rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle w_N, L(f_1) \rangle & \langle w_N, L(f_2) \rangle & \dots & \langle w_N, L(f_N) \rangle \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{pmatrix} = \begin{pmatrix} \langle w_1, g \rangle \\ \langle w_2, g \rangle \\ \vdots \\ \langle w_N, g \rangle \end{pmatrix} \tag{6}$$

The above equation can be written as

$$[Z]_{N \times N} \times [I]_{N \times 1} = [V]_{N \times 1}$$

$$Z_{ij} = \langle w_j, L(f_i) \rangle \quad \text{where; } i, j = 1, 2, \dots, N$$

$$I_{i1} = \alpha_i$$

$$V_{j1} = \langle w_j, g \rangle$$

In the matrix form the above equation may be written as,

$$[I] = [Z]^{-1}[V] \text{ [4][6][8]} \tag{7}$$

So, the unknown co-efficient matrix can be obtained by the matrix inversion method.

#### IV. LITERATURE SURVEY

##### 4.1. Background Of Study

The analysis of current distribution from the dipole antenna using numerical technique Method of moment is the most important problems in theory of antenna . Using Pocklington’s integral equation & weighting functions to find out the field equations is the highly calculative. Furthermore, this geometry may be useful for array of dipole antenna<sup>[2]</sup>.

##### 4.2 Objective Of The Study

The principle objectives of the research are depicted as follows.

1. To compute the current distribution of a dipole antenna using MATLAB.
2. Analyze driving point impedance or self impedance for the dipole antenna.
3. To achieve the current distribution of an array of dipole antenna & analyze the mutual impedance of the array.

##### 4.3 Scope Of The Study

The study will focus on the current distribution by using MOM which includes segmentation of the dipole length & current distribution along it<sup>[5]</sup>. The investigation will consider the surface current distribution which is zero at the two ends of the dipole antenna & we have to find out the conditions for the current in middle positions where the current is maximum for the dipole antenna.

##### 4.4 Significance Of Study

The analysis of uniform surface current distribution on dipole antenna may improve the understanding of complex dipole array designs. The literature survey of the research is hoped to determine the surface current distribution for dipole antenna & the self impedance. The study of survey might be of interests to related field of study and industry.

In [1], the point matching method is introduced where the POCKLINGTON’S INTEGRAL EQUATION(PIE) & HALLEN’S INTEGRAL EQUATION(HIE) is made equal<sup>[14]</sup>. The integral equation can be modified into the matrix form after formulating the equation in terms of Hallen’s integral equation and Pocklington's Integral Equations. For this purpose, N equal number of

segmentation are done & applying the point matching form of MOM. Pocklington's Integral Equation takes more time to be computed because of the complexity but gives better convergence at the feeding point.

In [2], the author explained an arbitrary load with the correlation of a receiving thin dipole wire in both reverberation chamber (RC) and anechoic chamber (AC). The Method of moments (MOM) is employed to calculate the current distributions in both cases along a thin dipole. For alternate current, polarization is radiated upon the dipole with a fixed incident angle; whereas in RC, this field is represented by an appropriate superposition of many incident plane waves with stochastic incident angle, polarization, phases.

In [3], two types of current distribution is generated for Galerkins and for point matching solution. Galerkins method is used when a similar function is used for both the basis and weighting functions. Here, the Entire-domain cosine function has been used by the author for comparing the results for both PIE and HIE. The Dirac-delta function as the weighting function & the pulse function is used as basis function in order to simplify HIE and PIE for transforming them to the matrix formation which are very easy to implement in MATLAB.

In [5], for solution of matrix equations, Gaussian elimination, LU decomposition, Condition Numbr method and Iterative Method has been used. The Iterative Method is applied in four different way namely, Conjugate Gradient, Bi-conjugate Gradient, Conjugate Gradient Squared, Bi-conjugate Gradient Stabilized and Stopping Criteria.

In [7], the author said that the more compact results can be obtained using integral equations, which are being solved using the Gaussian elimination method or similar techniques i.e., like LU decomposition method. It has been noticed that the matrix inversion is not an efficient technique, because it requires three times more formulations and thus three times longer execution time than the Gaussian Elimination method.

In [9], according to the author, a computer code should be prepared which uses the MOM for solving complex electromagnetic equations, usually requires experience and a lot of work. Often, these codes are prepared for certain sections of problems. Thus, there is no convergence guarantee and in many situations there does not exist a helpful measure of accuracy of the solution generated. Among all of these disadvantages, MOM is the much powerful technique available now-a-days for the formulation of electromagnetic field equations that involves linearity.

In [13], Sub-sectional basis functions have been described. The most frequent choice is a method of moment with sub-sectional basis functions. The most commonly used shapes for the elementary cells are the triangle and rectangular. Even though the triangle shapes are most flexible, rectangular cells involve simpler calculations. Three types of test functions are described by the author, namely rooftop functions with Galerkin, rooftop function with razor testing and two-dimensional pulses and point matching.

## V. FUTURE WORK

After finding the current matrix  $[I]$ , which is equal to some excitation voltage and the  $[Z]$  matrix of variable 'i' and 'j', the current elements can be obtained. In this thesis, further we will apply two types of basis function i.e. rooftop function and the pulse function for different length of the dipole antenna to observe the current waveform of the dipole antenna. After finding the current waveform for the dipole antenna, the current distribution of the dipole array will be start. The values of N for the rooftop function & the pulse function are different. We assume the feed voltage be 1volt. The other parameters have their usual values. Here, we assume 2N number of elements. Among the 2N elements all elements will be '0' except at the middle position as the dipole has the maximum current at the middle. Here, we will approximately take two points as the middle position for both the rooftop function and the pulse function.

## VI. CONCLUSION

In this paper, we have studied the literature on the dipole antenna and the numerical technique i.e., method of moments. The current for the basis function using MOM can be obtained through out the length of the dipole antenna. Computation of the integral equations for the Pocklington's Integral Equation is very complex using method of moment. The elements are generated from the co-efficient matrix  $[I]$  using the matrix inversion method. The overall implementation of the design will be performed in MATLAB.

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